Wedenesday, 8/23

Wednesday, August 23, 2023 7:37 AM

lesson2

NAME:______________________________

REVIEW OF INTEGRATION
MATH 16020

1 Antiderivatives

(a) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (where $n \neq -1$)

(b) $\int \frac{1}{x}\,dx = \ln|x| + C$

(c) $\int a\,dx = ax + C$ (where a is a constant)

(d) $\int \sin(x)\,dx = -\cos(x) + C$

(e) $\int \cos(x)\,dx = \sin(x) + C$

(f) $\int \sec^2(x)\,dx = \tan(x) + C$ $\quad \int \csc^2(x)\,dx = -\cot(x) + C$

(g) $\int \sec(x)\tan(x)\,dx = \sec(x) + C$ $\quad \int \csc(x)\cot(x)\,dx = -\csc(x) + C$

(h) $\int e^x\,dx = e^x + C$

(i) $\int af(x)\,dx = a\int f(x)\,dx$ (where a is a constant)

(j) $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

(k) $\int (f(x)-g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx$

Example 1. $\int \left(2e^x + \frac{5}{x} + \cot(x)(\csc(x) + \sin(x))\right)dx$

$5 \cdot \frac{1}{x}$

Antiderivative $= \int \left(2e^x + \frac{5}{x} + \cot(x)\csc(x) + \overbrace{\cot(x)\sin(x)}^{\cos(x)}\right)dx$

$\frac{\cos(x)}{\sin(x)} \cdot \sin(x)$

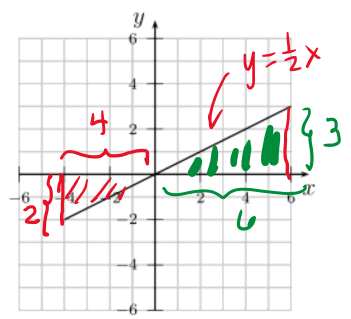
function $= 2e^x + 5\ln|x| - \csc(x) + \sin(x) + C$

2 Definite Integrals

- Indefinite integrals
  Antideriavtives (no limits on $\int$) are functions.
- Definite integrals (number limits on $\int$) are numbers.

The **geometric meaning** of $\int_a^b f(x)\,dx$ is the signed area between $y = f(x)$ and the x-axis $(y=0)$, from $x =$ a to $x =$ b.

Example 2. Find $\int_{-4}^{6} \frac{1}{2}x\,dx$ geometrically.

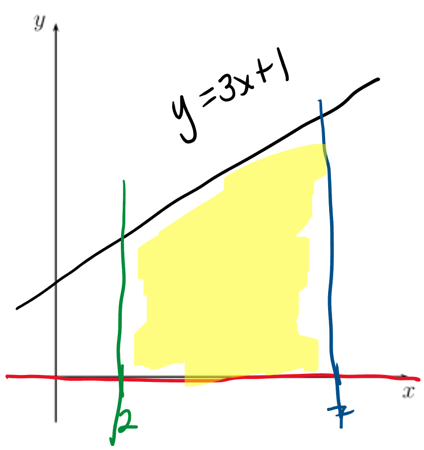


Red signed Area: $-\frac{1}{2}(4)(2) = -4$

Green signed Area: $+\frac{1}{2}(6)(3) = +9$

$$\int_{-4}^{6} \frac{1}{2}x\,dx = -4+9 = 5$$

Example 3. Set up a definite integral that represents the area of the region bounded by $y = 3x+1$, $\underbrace{y = 0}_{\text{x-axis}}$, $x = 2$, and $x = 7$.



$$\int_2^7 (3x+1)\,dx$$

3 The Fundamental Theorem of Calculus

Theorem 4 (Fundamental Theorem of Calculus, Part II). *If f is continuous on the interval $[a, b]$ and F is any antiderivative of f, then*

$F'(x) = f(x)$

i.e. $F(x) = \int f(x)\,dx$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Theorem 5 (Net Change Theorem). *If F is differentiable on the interval $[a, b]$, then*

FTC in disguise.

$$\int_a^b F'(x)\,dx = \underbrace{F(b) - F(a)}_{\text{net change}}$$

$\int_a^b \text{rate}\,dx = \text{net change}$

Ex: $v(t) = s'(t)$ deriv. position

$\int_a^b v(t)\,dt = \int_a^b s'(t)\,dt$

$= s(b) - s(a)$

Example 6. Evaluate the definite integrals.

(a) $\int_1^4 x^2\left(x + \frac{1}{\sqrt{x}}\right)dx$

$\frac{x^2}{\sqrt{x}} = \frac{x^2}{x^{1/2}} = x^{3/2}$

$= \int_1^4 (x^3 + x^{3/2})\,dx$

$= \frac{x^4}{4} + \frac{2}{5}x^{5/2}\Big|_1^4 = \left(\frac{4^4}{4} + \frac{2}{5}(4)^{5/2}\right) - \left(\frac{1^4}{4} + \frac{2}{5}(1)^{5/2}\right)$

$= \frac{1523}{20}$

(b) $\int_0^{16} \frac{\sqrt[4]{x^3}}{5}\,dx = \int_0^{16} \frac{1}{5}x^{3/4}\,dx$

$= \frac{1}{5}\cdot\frac{4}{7}x^{7/4}\Big|_0^{16} = \frac{4}{35}x^{7/4}\Big|_0^{16}$

$= \frac{4}{35}(16)^{7/4} - \frac{4}{35}(0)^{7/4}$

$= 512/35$

Example 7. At noon, a hot air balloon pilot begins to fill his balloon with air at a rate of

derivative

$$r(t) = 1000t^2$$

where t is measured in hours since noon and $r(t)$ is measured in cubic feet per hour.

rate

$V(t)$ volume at time t

$r(t) = V'(t)$

(a) How much air goes into the balloon between 1:00PM and 2:00PM?

$r(t) = V'(t)$

(a) How much air goes into the balloon between 1:00PM and 2:00PM?

$t=1 \quad t=2$

$$V(2)-V(1) = \int_1^2 V'(t)\,dt$$

$$= \int_1^2 r(t)\,dt = \int_1^2 1000t^2\,dt = \frac{1000t^3}{3}\Big|_1^2$$

$$= \frac{1000(2)^3}{3} - \frac{1000(1)^3}{3} = \frac{7000}{3} \text{ cubic feet}$$

(b) Approximately how many hours does it take to fill the hot air balloon? (Hot air balloons typically hold 77,000 cubic feet of air.) Round your answer to the nearest hundredth of an hour.

$t=0 \to t=T$ (time when filled)

Change in Vol = 77,000

About 6.14 hrs

$$\int_0^T V'(t)\,dt = 77{,}000$$

$$\int_0^T 1000t^2\,dt = 77{,}000$$

$$\frac{1000t^3}{3}\Big|_0^T = 77{,}000$$

$$\frac{1000T^3}{3} - \cancel{\frac{1000(0)^3}{3}} = 77{,}000$$

$$T^3 = 77{,}000\left(\frac{3}{1000}\right)$$

$$T^3 = 231$$

$$T = \sqrt[3]{231} \approx 6.14 \text{ hours}$$

Example 8. If $y' = \sin(x)$ and $y(0) = 5$, find $y\left(\frac{\pi}{6}\right)$.

$$\int_0^{\pi/6} y'\,dy = y\left(\tfrac{\pi}{6}\right) - y(0) \quad \text{Net change thm}$$

$$\int_0^{\pi/6} \sin(x)\,dx = y\left(\tfrac{\pi}{6}\right) - 5$$

$$-\cos(x)\Big|_0^{\pi/6}$$

$$-\cos\left(\tfrac{\pi}{6}\right) + \cos(0) = y\left(\tfrac{\pi}{6}\right) - 5$$

$$-\frac{\sqrt{3}}{2} + 1 = y\left(\tfrac{\pi}{6}\right) - 5$$

$$+5 \qquad +5$$

$$y\left(\tfrac{\pi}{6}\right) = 6 - \frac{\sqrt{3}}{2}$$